VITEEE 2021 Solutions May 28 - Slot 2

Ques: The equation of the normal to the hyperbola 25x2- 64y2 = 1600 at the point (16, 5√5) is: 1.√5x + 4y = 89 2.√5x + 4y = 39 3.4x + √5y = 89 4.4x + √5y = 39

Solution.
The equation of the hyperbola is:
$$ 25x^2 - 64y^2 = 1600 $$

Dividing by 1600 to normalize it:
$$ \frac{x^2}{64} - \frac{y^2}{25} = 1 $$

This means that the hyperbola is of the form:
$$ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 $$
where $a^2 = 64$ (so $a = 8$) and $b^2 = 25$ (so $b = 5$).

The equation for the tangent to this hyperbola at any point (x, y) is given by:
$$ \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1 $$
Where (x_1, y_1) is the point of tangency.

Given that the point is $(16, 5\sqrt{5})$, plugging in the values:
$$ \frac{x(16)}{64} - \frac{y(5\sqrt{5})}{25} = 1 $$
$$ \frac{x}{4} - \frac{y\sqrt{5}}{5} = 1 $$
$$ x - 4y\sqrt{5} = 4 $$
This is the equation of the tangent.

Now, for the normal:
The equation of the normal to the hyperbola at the point (x_1, y_1) is:
$$ ax\cdot x_1 - by\cdot y_1 = (a^2 - b^2) $$

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Plugging in the given point and our values for a and b:
$$ 8x\cdot 16 - 5y\cdot 5\sqrt{5} = 64 - 25 $$
$$ 128x - 25y\sqrt{5} = 39 $$
Dividing everything by 32 to simplify:
$$ 4x - \frac{25\sqrt{5}y}{32} = \frac{39}{32} $$

Multiplying everything by 32 to clear out the denominator:
$$ 128x - 25\sqrt{5}y = 39 $$

This is the equation of the normal. Now, rearrange terms to match the given options:
$$ 4x + \frac{25y}{\sqrt{5}} = \frac{39}{\sqrt{5}} $$
Multiplying everything by $\sqrt{5}$:
$$ 4x\sqrt{5} + 25y = 39\sqrt{5} $$
$$ 4x + \sqrt{5}y = 39 $$

The correct equation matches option:
3. $ 4x + \sqrt{5}y = 39 $

Ques: Which of the following is a tautology?
1.(P v Q)↔(P v (P↔(Q↔R)))
2.(P v Q)↔(Q v (P↔(Q↔R)))
3.(P v Q)↔(P v (Q↔(R↔Q)))
4.(P v Q)↔(Q v (Q↔(R↔Q)))

Solution.
A tautology is a statement that is always true regardless of the truth values of its individual propositions.

Let's evaluate the given options:

1. $ (P \vee Q) \leftrightarrow (P \vee (P \leftrightarrow (Q \leftrightarrow R))) $
- When P is true, regardless of the values of Q and R, the left side (P v Q) is true. For the right side, if P is true, then $ P \vee (P \leftrightarrow (Q \leftrightarrow R)) $ is always true regardless of the biconditional.

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- When P is false and Q is true, the left side is still true. On the right side, the biconditional's truth value will determine the overall truth value, making it not always true.
This is not a tautology.

2. \((P \vee Q) \leftrightarrow (Q \vee (P \leftrightarrow (Q \leftrightarrow R))) \)
- When Q is true, the whole statement is true regardless of P and R.
- When Q is false, and P is true, the biconditional's value will determine the truth of the overall statement.
This is not a tautology.

3. \((P \vee Q) \leftrightarrow (P \vee (Q \leftrightarrow (R \leftrightarrow Q))) \)
- When P is true, the whole statement is true regardless of Q and R.
- When P is false and Q is true, the biconditional \((Q \leftrightarrow (R \leftrightarrow Q)) \) is always true because Q biconditional with anything involving Q will always be true.
This is a tautology.

4. \((P \vee Q) \leftrightarrow (Q \vee (Q \leftrightarrow (R \leftrightarrow Q))) \)
- Similar to the 3rd option, if Q is true, then the statement is always true regardless of P and R.
This is a tautology.

Therefore, the tautologies are:
3. \((P v Q) \leftrightarrow (P v (Q \leftrightarrow (R \leftrightarrow Q))) \)
4. \((P v Q) \leftrightarrow (Q v (Q \leftrightarrow (R \leftrightarrow Q))) \)

Ques: The domain for which the function f(x) = 2x2-1 andg (x) = 1 - 3x are equal is
1.{2}
2.{½}
3.-2, ½}
4.{-2}

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Solution.
Given two functions:

$$f(x) = 2x^2 - 1$$

$$g(x) = 1 - 3x$$

To find the domain for which they are equal, we equate $f(x)$ to $g(x)$.

$$2x^2 - 1 = 1 - 3x$$

Simplifying and collecting like terms:

$$2x^2 + 3x - 2 = 0$$

Now, we will solve for x.

To factor this quadratic equation, look for two numbers that multiply to give $2 \times (-2) = -4$ and add up to 3. Those numbers are 4 and -1.

The equation can be expressed as:

$$2x^2 + 4x - x - 2 = 0$$

Group the terms:

$$2x(x + 2) - 1(x + 2) = 0$$

Factor out the common term:

$$(2x - 1)(x + 2) = 0$$

From this, we get:

$$2x - 1 = 0$$ or $$x + 2 = 0$$

Solving, we get:

$$x = \frac{1}{2}$$ or $$x = -2$$

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So, the domain for which $f(x)$ and $g(x)$ are equal is:
$x = \frac{1}{2}$ and $x = -2$

The correct answer is:
3. $\{ -2, \frac{1}{2} \}$

Q: The angle between the curves y = x3and y = x5at x = 0 is
1.π/2
2.0
3.π/3
4.π/4

Solution.
To find the angle between the curves, we need to find the angle between their tangents at the given point.

First, let's find the slope of the tangent (i.e., the derivative) for each curve at $x = 0$.

For $y = x^3$:
$$\frac{dy}{dx} = 3x^2$$
At $x = 0$, $\frac{dy}{dx} = 0$.

For $y = x^5$:
$$\frac{dy}{dx} = 5x^4$$
At $x = 0$, $\frac{dy}{dx} = 0$.

Both tangents are horizontal at $x = 0$. Therefore, the angle between them is 0 degrees or 0 radians.

So, the correct answer is:
2. 0

Q: As mass A increases. Which of the following quantities related to a nucleus do not change?
1. Volume
2. Density
3. Mass

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4. **Binding Energy**

Solution.
The question relates to nuclear properties as the mass number, \(A \), increases.

1. **Volume**: As \(A \) (the number of nucleons) increases, the volume of the nucleus also increases because there are more protons and neutrons.

2. **Density**: The nuclear density is approximately constant for all nuclei. This suggests that the nucleus is uniformly packed and does not depend significantly on the number of nucleons.

3. **Mass**: As \(A \) increases, the mass of the nucleus also increases because there are more nucleons.

4. **Binding Energy**: The binding energy is the energy required to disassemble a nucleus into its constituent protons and neutrons. It varies with the number of nucleons and is not constant.

So, the only quantity that does not change as mass \(A \) increases is:
2. **Density**.

Ques: A tetrapeptide possesses ______________ peptide bonds and can be ______ into amino acids fragments
1.4; hydrolysed
2.3; hydrolysed
3.4; denatured
4.4; denatured

Solution.
A tetrapeptide is composed of four amino acids. When amino acids are joined together through peptide bonds, the number of peptide bonds will always be one less than the number of amino acids. This is because each bond connects two amino acids.

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1. For a dipeptide (two amino acids), there is 1 peptide bond.
2. For a tripeptide (three amino acids), there are 2 peptide bonds.
3. For a tetrapeptide (four amino acids), there are 3 peptide bonds.

Furthermore, peptide bonds can be broken down through hydrolysis. Denaturation typically refers to the loss of the three-dimensional structure of proteins (like when an egg is fried), but it doesn't break the peptide bonds.

So, the correct answer is:
2. **3; hydrolysed**.

Ques: Which of the following statements is true?
1. The wavelength of light emitted by LED depends on the applied voltage across the LED.
2. LED emits light when it is reverse biased
3. The V-I characteristics of an LED is similar to that of a silicon p-n junction diode
4. LED is made of a thin layer of lightly doped semiconductor material

Solution.
Let's examine each statement:

1. **The wavelength of light emitted by LED depends on the applied voltage across the LED.**
   - This statement is true to a certain extent. The color (or wavelength) of the light emitted by an LED is primarily determined by the energy bandgap of the semiconducting material it's made from. However, there can be slight variations in the emitted wavelength based on the current (which is related to the applied voltage) through the LED. At high current levels, the wavelength can shift slightly. But in common usage, the color of an LED is generally determined by its material, not the voltage applied.

2. **LED emits light when it is reverse biased.**

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- This statement is false. LEDs emit light when they are forward biased. In reverse bias, they don't emit light and can get damaged if the reverse voltage exceeds the breakdown voltage.

3. **The V-I characteristics of an LED is similar to that of a silicon p-n junction diode.**
   - This statement is not completely accurate. While both LED and a silicon p-n junction diode show a knee voltage and begin conducting in forward bias, an LED starts emitting light after its threshold voltage. Furthermore, the "knee" or threshold voltage for an LED is typically higher than that for a silicon diode.

4. **LED is made of a thin layer of lightly doped semiconductor material.**
   - This statement is not entirely accurate. An LED is made from a p-n junction of semiconducting materials. The doping levels can vary, but to say it's just a "thin layer of lightly doped semiconductor material" oversimplifies the structure.

Of the given options, the closest to being accurate is the first statement, but it's not completely true without qualifications. So, strictly speaking: None of the given statements are unequivocally true without reservations.

Ques: The resistance of a copper wire is 1.05 Ω at 20 degree C. What will be resistance at 0 degree C? The temperature coefficient of resistivity of copper is0.00393 degree C-1. 1.0.52 Ω 2.0.31 Ω 3.0.89 Ω 4.0.97 Ω

Solution.
To determine the resistance at 0°C, given the resistance at 20°C, we use the formula that relates resistance, temperature, and the temperature coefficient of resistivity, α:

$$R_T = R_0 (1 + \alpha (T - T_0))$$

Where:

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- R_T is the resistance at temperature T.
- R_0 is the resistance at the reference temperature T_0.
- α is the temperature coefficient of resistivity.

Given:
- R_0 (resistance at 20°C) = 1.05 Ω.
- T_0 = 20°C.
- T = 0°C.
- α = 0.00393 °C^(-1).

We are to find R_T, the resistance at 0°C. Plugging in the given values:

$$R_T = 1.05 (1 + 0.00393 (0 - 20))$$

$$R_T = 1.05 (1 - 0.0786)$$
$$R_T = 1.05 \times 0.9214$$
$$R_T \approx 0.96747$$

Rounding, we get:
$$R_T \approx 0.97 \Omega$$

So, the correct answer is:
4. **0.97 Ω**.

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