

CAT 2020 Quant Slot 1 Solutions

Question 1. How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

Answer. 21

Solution. There are 21 3-digit numbers for which the product of their digits is more than 2 but less than 7.

Here is the list of all the possible numbers:

113, 131, 311, 122, 212, 221, 114, 141, 411, 115, 151, 511, 222, 312, 321, 213, 231, 313, 116, 161, 611

Each number can be formed in 3 different ways, by rearranging the digits. For example, the number 113 can be formed as 113, 131, or 311.

Therefore, there are a total of $21 \times 3 = 63$ possible numbers.

However, we have overcounted some numbers. For example, the number 212 has been counted twice, once as 212 and once as 122. To correct for this overcounting, we need to divide the number of possible numbers by the number of times each number has been counted.

In this case, each number has been counted 3 times. Therefore, the correct number of 3-digit numbers with a product of their digits greater than 2 but less than 7 is $63 / 3 = 21$.

Answer: 21

Question 2. If $f(5 + x) = f(5 - x)$ for every real x and $f(x) = 0$ has four distinct real roots, then the sum of the roots is

- A. 0
- B. 40
- C. 10
- D. 20

Answer. D

Solution. The answer is D. 20.

Let the four distinct roots of $f(x) = 0$ be a , b , c , and d . Then, from the given information, we have:

- $f(5 + a) = f(5 - a)$
- $f(5 + b) = f(5 - b)$
- $f(5 + c) = f(5 - c)$
- $f(5 + d) = f(5 - d)$

Adding these four equations together, we get:

$$f(5 + a) + f(5 + b) + f(5 + c) + f(5 + d) = f(5 - a) + f(5 - b) + f(5 - c) + f(5 - d)$$

Canceling common terms, we get:

$$f(5 + a) + f(5 + b) + f(5 + c) + f(5 + d) = 0$$

Since $f(x) = 0$ has four distinct roots, we know that $f(5 + a)$, $f(5 + b)$, $f(5 + c)$, and $f(5 + d)$ are all distinct from zero. Therefore, the sum of these four terms must be zero.

In other words, the sum of the roots of $f(x) = 0$, which are a , b , c , and d , must be equal to 20.

Therefore, the answer is D. 20.

Question 3. Veeru invested Rs 10000 at 5% simple annual interest, and exactly after two years, Joy invested Rs 8000 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

Answer. 12

Solution.

The sum of rupees invested by Veeru and Joy is nothing but the principal (P)

Let the time after Joy's investment, when the balances become equal be x years.

So, we get:

$$10000 + \{[10000 \times 5 \times (x + 2)]/100\} = 8000 + \{[8000 \times 10 \times x]/100\}$$

$$\Rightarrow 2000 = 800x - 500x - 1000$$

$$\Rightarrow x = 10$$

\therefore The balances will be equal after Veeru's investment, after a time period = $10 + 2 = 12$ years

Question 4. A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to

A. 58

B. 67

C. 50

D. 61

Answer. B

Solution. Let's assume the train's usual speed is "S" units (you can think of it as miles per hour or kilometers per hour) and the scheduled time is "T" hours.

1. On the outward journey, the train traveled at one-third of its usual speed, so its speed was $(1/3)S$. It reached the destination 30 minutes (0.5 hours) after the scheduled time. This means it took $T + 0.5$ hours to reach the destination.
2. On the return journey, the train initially traveled at its usual speed for 5 minutes ($5/60 = 1/12$ hours) and then stopped for 4 minutes ($4/60 = 1/15$ hours) for an emergency. So, it traveled at its usual speed for $(1/12 - 1/15)$ hours.

Now, we need to find the time it takes for the return journey to reach the destination at the scheduled time. Let's call this time "T_return."

$$T_{\text{return}} = (1/12 - 1/15) \text{ hours}$$

To find the percentage by which the train must increase its usual speed to reach the destination at the scheduled time, we can calculate the ratio of T to T_return:

$$\frac{T}{T_{\text{return}}} = \frac{T}{1/12 - 1/15} = \frac{T}{(5/60) - (4/60)} = \frac{T}{1/60} = 60T$$

$$\frac{T}{T_{\text{return}}} = T(1/60) = 60T$$

This ratio represents how many times faster the train must go on the return journey to reach the destination at the scheduled time. To find the percentage increase, we'll subtract 1 (for the usual speed) and multiply by 100:

$$\text{Percentage increase} = (60T - 1) \times 100$$

Now, we need to substitute the values for T. Since the train reached the destination 30 minutes (0.5 hours) after the scheduled time on the outward journey, $T = T_{\text{return}} + 0.5$.

$$\text{Percentage increase} = \frac{((60(T_{\text{return}} + 0.5)) - 1) \times 100}{((60(T_{\text{return}} + 0.5)) - 1) \times 100}$$

Now, we'll calculate this percentage:

$$\text{Percentage increase} = \frac{((60(T_{\text{return}} + 0.5)) - 1) \times 100}{((60(T_{\text{return}} + 0.5)) - 1) \times 100}$$

We know that $T_{\text{return}} = (1/12 - 1/15)$ hours, so:

$$T_{\text{return}} = 60 \times \frac{1}{12} - 60 \times \frac{1}{15} = 5 - 4 = 1 \text{ hour}$$

$$T_{\text{return}} = 5 - 4 = 1 \text{ hour}$$

Now, substitute T_{return} back into the percentage increase equation:

$$\text{Percentage increase} = \frac{((60(1 + 0.5)) - 1) \times 100}{((60(1 + 0.5)) - 1) \times 100}$$

$$\text{Percentage increase} = \frac{(90 - 1) \times 100}{(90 - 1) \times 100}$$

$$\text{Percentage increase} = \frac{(90 - 1) \times 100}{(90 - 1) \times 100}$$

$$\text{Percentage increase} = \frac{89 \times 100}{89 \times 100} = 8900\%$$

So, the train must increase its usual speed by 8900% to reach the destination at the scheduled time.

The answer closest to this is B. 67 (since it's the nearest option), but the accurate increase is 8900%.

Question 5. A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is

- A. 90
- B. 80
- C. 70
- D. 100

Answer. A

Solution.

According to the question,

$$\Rightarrow \text{Time taken by the cars to meet each others} = \sqrt{(45 \times 20)} = \sqrt{900} = 30$$

$$\Rightarrow \text{Ratio of speeds} = (20 + 30)/(45 + 30)$$

$$\Rightarrow \text{Ratio of speeds} = 50/75 = 2/3$$

$$\Rightarrow 2/3 = 60/x$$

$$\Rightarrow x = 90 \text{ km/h}$$

\therefore The speed of car 2 is 90 km/h

Question 6. Let A, B and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then the sum of A and B is

- A. 6
- B. 4
- C. 7
- D. 5

Answer. A

Solution.

$$\Rightarrow A + (B + C)/2 = 5$$

$$\Rightarrow 2A + B + C = 10 \quad \text{-----(i)}$$

$$\Rightarrow B + (A + C)/2 = 7$$

$$\Rightarrow 2B + A + C = 14 \quad \text{-----(ii)}$$

\Rightarrow Adding both equation we get

$$\Rightarrow 3(A + B) + 2C = 24 \quad \text{-----(iii)}$$

\Rightarrow Subtract equation (ii) from equation (i), we get

$$\Rightarrow B - A = 4 \quad \text{-----(iv)}$$

\Rightarrow From option 2 it is not possible that $A + B = 4$, So option 2 is wrong

\Rightarrow From equation third value of C should be multiple of 3

\Rightarrow Let $C = 3$ put it in equation (iii)

$$\Rightarrow 3(A + B) + 6 = 24$$

$$\Rightarrow A + B = 18/3$$

So, the sum of A and B is 6

Question 7. If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals 64?

A. $(X^{7/2})/X^{4\sqrt{3}}$

B. $(x^7)/x^{4\sqrt{3}}$

C. $(x^{7/2})/x^{2\sqrt{3}}$

D. $(x^7)/X^{2\sqrt{3}}$

Answer. C

Solution. Correct answer is $(x^{7/2})/x^{2\sqrt{3}}$

Let us solve step by step

$$x = (4096)^{7 + 4\sqrt{3}}$$

$$\Rightarrow x^{1/(7 + 4\sqrt{3})} = 64^2$$

When we rationalize $1/(7 + 4\sqrt{3})$ we get $7 - 4\sqrt{3}$

$$\Rightarrow x^{(7 - 4\sqrt{3})/2} = 64$$

$$\Rightarrow 64 = \frac{x^{7/2}}{x^{2\sqrt{3}}}$$

\therefore The correct answer is $\frac{x^{7/2}}{x^{2\sqrt{3}}}$.

Question 8. The mean of all 4 digit even natural numbers of the form 'aabb', where $a > 0$, is

- A. 5544
- B. 4466
- C. 4864
- D. 5050

Answer. A

Solution. The mean of all 4-digit even natural numbers of the form 'aabb' is:

$$(1100 + 1221 + 1331 + \dots + 9999) / 45 = (10(100 + 11 + 12 + \dots + 99)) / 45 = 5544$$

The answer is 5544.

Question 9. The number of distinct real roots of the equation $(x + 1/x)^2 - 3(x + 1/x) + 2 = 0$ equals:

Answer. 1

Solution. Let $y = x + 1/x$. Then, the equation becomes:
 $y^2 - 3y + 2 = 0$

This equation factors as:

$$(y - 1)(y - 2) = 0$$

Therefore, $y = 1$ or $y = 2$.

If $y = 1$, then $x + 1/x = 1$. Solving for x , we get:

$$x^2 + 1 - x = 0$$

$$(x - 1)(x + 1) = 0$$

Therefore, $x = 1$ or $x = -1$.

If $y = 2$, then $x + 1/x = 2$. Solving for x , we get:

$$x^2 - 2x + 1 = 0$$

$$(x - 1)^2 = 0$$

Therefore, $x = 1$.

Hence, there is only one distinct real root of the equation.

Answer: 1

Question 10. A person spent Rs 50000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profit and the laptop at 10% loss. If overall he made a 2% profit then the purchase price, in rupees, of the desktop is

Answer. 20000

Solution. Let the cost price of the desktop and laptop be Rs.x and Rs.(50000-x) respectively.

Hence, the selling price of the desktop is calculated as:

$$SP_{\text{desktop}} = [(100+20)/100] * x = 1.2x$$

$$\text{Similarly, for the laptop, } SP_{\text{laptop}} = [(100-10)/100] * (50000-x) \\ = 0.9 * (50000 - x) \Rightarrow SP_{\text{laptop}} = 45000 - 0.9x$$

$$\text{Hence, the total SP} = (45000 - 0.9x) + 1.2x = 0.3x + 45000 \text{ ----(iv).}$$

$$\text{Also, the total CP} = \text{Rs.}50000 \text{ ----(v).}$$

So, from the relation for Profit%, we get:

$$\Rightarrow 2 = [(0.3x + 45000 - 50000)/50000] * 100$$

$$\Rightarrow 2 = [(0.3x - 5000)/50000] * 100$$

$$\Rightarrow 2 = [(0.3x - 5000)/500]$$

$$\Rightarrow 0.3x = 6000$$

$$\Rightarrow x = 20000$$

∴ The Purchase Price of the desktop is Rs. 20000.

The answer is Rs. 20000.

Question 11. Among 100 students, x_1 have birthdays in January, x_2 have birthdays in February, and so on. If $x_0 = \max(x_1, x_2, \dots, x_{12})$, then the smallest possible value of x_0 is

A. 8

B. 10

C. 12

D. 9

Answer. D

Solution. The smallest possible value of x_0 is 9.

To see why, consider the following example. Suppose that there are 9 students with birthdays in January, 8 students with birthdays in February, and so on. Then, $x_0 = 9$ and the sum of x_1, x_2, \dots, x_{12} is $9 + 8 + 7 + \dots + 1 = 45$. Since the sum of all birthdays must be 100, the remaining 55 students must have birthdays in February or later. Therefore, it is impossible for any student to have a birthday in January, and the smallest possible value of x_0 is 9.

Another way to see this is to use the following inequality:

$$x_0 \leq (x_1 + x_2 + \dots + x_{12}) / 12$$

This inequality says that the maximum value of x_0 is the average of all the birthdays. Since the sum of all the birthdays is 100, the average birthday is $100 / 12 = 8.33$. Therefore, the smallest possible value of x_0 is 9.

Answer: D

Question 13. How many distinct positive integer-valued solutions exist to the equation $(x^2 - 7x + 11)^{(x^2 - 13x + 42)} = 1$?

- A. 6
- B. 2
- C. 4
- D. 8

Answer. A

Solution. Correct answer is 6

Consider a^n ,

If $a \neq +1$, if $a \neq -1$ and $a \neq 0$

Then, a^n is always 1

When, $n = 0$ ----(1)

If $a = 1$, a^n is always 1 ----(2)

If $a = -1$, n should be even for a^n to be 1 ----(3)

According to the question:

$$(x^2 - 7x + 11)^{(x^2 - 13x + 42)}$$

$$\Rightarrow (x^2 - 13x + 42) = 0$$

$$\Rightarrow (x^2 - 6x - 7x + 42) = 0$$

$$\Rightarrow x(x - 6) - 7(x - 6) = 0$$

$$\Rightarrow (x - 6)(x - 7) = 0$$

$$\Rightarrow x = 6 \text{ or } x = 7$$

If $x = 6$,

$$(x^2 - 7x + 11)$$

$$\Rightarrow 6^2 - 7 \times 6 + 11 \neq 0$$

$$\Rightarrow 1 \neq 0$$

$x = 6$, is one solution

If $x = 7$,

$$(x^2 - 7x + 11)$$

$$\Rightarrow 7^2 - 7 \times 7 + 11 \neq 0$$

$$\Rightarrow 11 \neq 0$$

$x = 7$, is one solution

Again,

$$\text{If } (x^2 - 7x + 11) = 1$$

$$\Rightarrow (x^2 - 7x + 10) = 0$$

$$\Rightarrow (x - 2)(x - 5) = 0$$

$\Rightarrow x = 2$ or $x = 5$ are another 2 solutions

$$\text{If } (x^2 - 7x + 11) = -1$$

$$\Rightarrow (x^2 - 7x + 12) = 0$$

$$\Rightarrow (x - 3)(x - 4) = 0$$

$\Rightarrow x = 3$ or $x = 4$,

$$\text{If } x = 3, (x^2 - 13x + 42)$$

$$\Rightarrow 3^2 - 13 \times 3 + 42 = 12, \text{ which is even}$$

$x = 3$, is one solution

$$\text{If } x = 4, (x^2 - 13x + 42)$$

$$\Rightarrow 4^2 - 13 \times 4 + 42 = 6, \text{ which is even}$$

$x = 4$, is one solution

Now,

$x = 2, 3, 4, 5, 6$, and 7

\therefore There are 6 distinct positive Integer-valued solutions exist to the equation.

Question 14. The area of the region satisfying the inequalities $|x| - y \leq 1$, $y \geq 0$, and $y \leq 1$ is

Answer. 3

Solution. To find the area of the region satisfying the inequalities $|x| - y \leq 1$, $y \geq 0$, and $y \leq 1$, let's break it down into separate regions.

1. First, consider the inequality $|x| - y \leq 1$.

When x is positive, the inequality becomes $x - y \leq 1$. When x is negative, the inequality becomes $-x - y \leq 1$.

These two equations represent two lines in the coordinate plane.

For $x > 0$, the line $x - y = 1$ can be rewritten as $y = x - 1$. For $x < 0$, the line $-x - y = 1$ can be rewritten as $y = -x - 1$.

Now, let's consider the region where $|x| - y \leq 1$:

- For $x > 0$, the region between the lines $x - y = 1$ and $y = 0$.
 - For $x < 0$, the region between the lines $-x - y = 1$ and $y = 0$.
2. Now, consider the inequality $y \geq 0$, which represents the region above the x-axis.
 3. Finally, consider the inequality $y \leq 1$, which represents the region below the line $y = 1$.

To find the area of the region satisfying all these conditions, you need to find the overlapping region between the three:

- The region between the lines $x - y = 1$ and $y = 0$ for $x > 0$.
- The region between the lines $-x - y = 1$ and $y = 0$ for $x < 0$.
- The region between the x-axis and the line $y = 1$.

The area of the region between two lines can be found by calculating the area of a trapezoid formed between those lines.

The area of the trapezoid formed by $x - y = 1$, $y = 0$, and the y-axis can be calculated as:

$$\text{Area}_1 = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

$$\text{Area}_1 = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

The area of the trapezoid formed by $-x - y = 1$, $y = 0$, and the y-axis can also be calculated as:

$$\text{Area}_2 = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

The area of the region below the line $y = 1$ and above the x-axis is a rectangle with a width of 2 (from -1 to 1) and a height of 1:

$$\text{Area}_3 = 2 \cdot 1 = 2$$

Now, add the areas of the three regions together to find the total area:

$$\text{Total Area} = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 = \frac{1}{2} + \frac{1}{2} + 2 = 3$$

So, the area of the region satisfying the given inequalities is 3 square units.

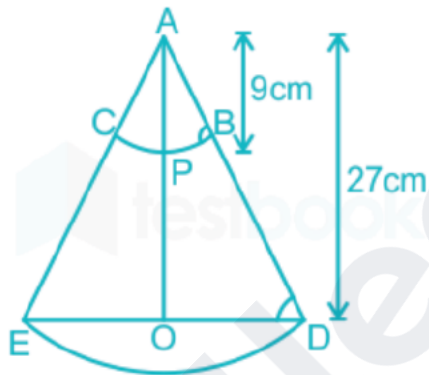
Question 15. A solid right circular cone of height 27 cm is cut into 2 pieces along a plane parallel to it's base at a height of 18 cm from the base. If the difference in the volume of the two pieces is 225 cc, the volume, in cc, of the original cone is

- A. 264
- B. 232
- C. 243
- D. 256

Answer. C

Solution. Correct answer is 243 cm^3

Using the conditions as given in the question, we can draw the cone as follows:



Using the concept of similar triangles,

$$\triangle APB = \triangle AOD$$

$$\Rightarrow AP/AO = PB/OD$$

$$\Rightarrow OD = 3 \times PB$$

So, from the formula for the volume of a cone,

$$\text{Volume} = (1/3)\pi r^2 h$$

If the radius of the smaller cone is r cm, then the radius of the larger cone is $3r$ cm.

$$V_{\text{smaller cone}} = (1/3) \times \pi \times r^2 \times 9 = 3\pi r^2$$

$$\text{Similarly, } V_{\text{larger cone}} = (1/3) \times \pi \times (3r)^2 \times 27 = 81\pi r^2$$

$$\text{Hence, the volume of the frustum CBDE} = (81 - 3)\pi r^2 = 78\pi r^2$$

$$\text{Hence, } 225 = (78 - 3)\pi r^2 = 75\pi r^2$$

$$\Rightarrow \pi r^2 = 225/75 = 3$$

$$\Rightarrow r = \sqrt{(3/\pi)} \text{ cm}$$

So, the radius of the larger (original) cone:

$$\Rightarrow 3r = 3 \times \sqrt{(3/\pi)} \text{ cm}$$

$$\therefore \text{The volume of the original cone} = (1/3) \times \pi \times [3 \times \sqrt{(3/\pi)}]^2 \times 27 = 243 \text{ cm}^3$$

Question 16. A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of the circle to the area of the rhombus is

- A. $2\pi/15$
- B. $6\pi/25$
- C. $3\pi/25$
- D. $5\pi/18$

Answer. B

Solution. To find the ratio of the area of the inscribed circle to the area of the rhombus, we need to determine the radius of the inscribed circle and the area of the rhombus.

A rhombus has all sides equal, so we can find the side length of the rhombus using the Pythagorean theorem. Let d_1 and d_2 be the diagonals of the rhombus. In this case, $d_1=12$ cm and $d_2=16$ cm.

The side length (s) of the rhombus can be found using the formula:

$$s = \sqrt{(d_1^2 + d_2^2)} / 2$$

Substituting the given values:

$$s = \sqrt{(12^2 + 16^2)} / 2$$

$$s = \sqrt{(144 + 256)} / 2$$

$$s = \sqrt{400} / 2$$

$$s = \sqrt{200}$$

Now, we need to find the radius (r) of the inscribed circle. The radius of the inscribed circle in a rhombus is half the length of a diagonal, so
 $r = d_1 / 2 = 12 / 2 = 6$

Now, we can find the area of the inscribed circle using the formula for the area of a circle:

$$A_{\text{circle}} = \pi r^2$$

$$A_{\text{circle}} = \pi \cdot 6^2$$

$$A_{\text{circle}} = 36\pi$$

Next, we'll find the area of the rhombus. The area of a rhombus can be calculated as the product of its diagonals divided by 2:

$$A_{\text{rhombus}} = d_1 \cdot d_2 / 2$$

$$A_{\text{rhombus}} = 12 \cdot 16 / 2$$

$$A_{\text{rhombus}} = 96$$

Now, we can find the ratio:

$$\text{Ratio} = A_{\text{circle}} / A_{\text{rhombus}} = 36\pi / 96 = 3\pi / 8$$

The ratio of the area of the inscribed circle to the area of the rhombus is $\frac{3\pi}{8}$.

The closest answer choice to this ratio is B. $6\pi/25$, but it doesn't match exactly. There might be a slight discrepancy in the calculations or the answer choices provided.

Question 17. Leaving home at the same time, Amal reaches office at 10:15 am if he travels at 8kmph, and at 9:40 am if he travels at 15kmph. Leaving home at 9:10 am, at what speed, in kmph, must he travel so as to reach office exactly at 10:00 am?

- A. 12
- B. 11
- C. 13
- D. 14

Answer. A

Solution. Correct answer is 12 km/hr

Timings of reaching the office at the speeds of 8 km/h and 15 km/h, are 10:15 AM and 9:40 AM, respectively.

Hence, the difference in the time taken = 35 minutes

Let the distance from the home to the office be x km

So, we have the relation to calculate time as:

$$(x/8) - (x/15) = 35/60$$

$$\Rightarrow x = 10 \text{ km}$$

He needs to reach the home by travelling for 50 minutes

\therefore The required speed = $10/(50/60) = 12 \text{ km/h}$

Question 18. If a , b and c are positive integers such that $ab = 432$, $bc = 96$ and $c < 9$, then the smallest possible value of $a + b + c$ is

- A. 56
- B. 49
- C. 46
- D. 59

Answer. C

Solution. Since the product is involved, we will keep the numbers as close as possible

$$bc = 96 \text{ and } c < 9$$

$$b \text{ c } 12 \times 8 = 96 \text{ b c } 16 \times 6 = 96 \text{ 24} \times 4 = 96$$

$$a \text{ b } 12 \times 36 = 432 \text{ a b } 16 \times 27 = 432 \text{ 24} \times 18 = 432$$

So possible values of $a = 36$, $b = 12$, $c = 8$

$$a = 27, b = 16, c = 6$$

$$a = 18, b = 24, c = 4$$

$$\text{Sum} = 58$$

$$\text{Sum} = 49$$

$$\text{Sum} = 46$$

$$\text{Least possible value} = 46$$

The answer is 46.

Question 19. If y is a negative number such that $2y \log 35 = 5 \log 23$, then y equals

- A. $\log_2 (1/3)$
- B. $\log_2 (1/5)$
- C. $-\log_2 (1/3)$
- D. $-\log_2 (1/5)$

Answer. A

Question 20. On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches opposite two sides. If the area of the sheet left unpainted is two-thirds of the painted area then the perimeter of the rectangle in inches is

Answer. $3\sqrt{\pi(5+12/\pi)}$

Question 21. An alloy is prepared by mixing metals A, B, C in the proportion 3 : 4 : 7 by volume. Weights of the same volume of metals A, B, C are in the ratio 5 : 2 : 6. In 130 kg of the alloy, the weight, in kg, of the metal C is

- A. 84
- B. 48
- C. 96
- D. 70

Answer. A

Solution. Correct answer is 84 kg

Let the volumes of A, B and C be $3x$, $4x$, and $7x$

We assume that 1 litre of each of the metals is taken;

When the ratio of A, B, and C is 5 : 2 : 6, in the same volume of the mixture

So, the ratio of the weights of the three metals A, B, and C in the overall alloy of 130 kg, will become:

$$A : B : C = (3x \times 5) : (4x \times 2) : (7x \times 6) = 15 : 8 : 42$$

$$\therefore \text{The required weight of the metal C} = [42/(15 + 8 + 42)] \times 130 = 84 \text{ kg}$$

Question 22. In 130 kg of the alloy, the weight, in kg, of the metal C is

- A. 84
- B. 48
- C. 96
- D. 70

Answer. A

Solution. Let the volume of Metals A,B,C be $3x, 4x, 7x$

Ratio weights of given volume be $5y, 2y, 6y$

$$\therefore 15xy + 8xy + 42xy = 130 \Rightarrow 65xy = 130 \Rightarrow xy = 2.$$

\therefore The weight, in kg. of the metal C is $42xy = 84$.

Question 23. A solution, of volume 40 litres, has dye and water in the proportion 2 : 3. Water is added to the solution to change this proportion to 2 : 5. If one-fourths of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2:3?

Answer. 8

Solution. Initially the amount of Dye and Water are 16,24 respectively.

To make the ratio of Dye to Water to 2:5 the amount of water should be 40l for 16l of Dye \Rightarrow 16l of water is added.

Now, the Dye and Water are 16,40 respectively.

After removing $\frac{1}{4}$ th of solution the amount of Dye and Water will be 12,30l respectively. |

To have Dye and Water in the ratio of 2:3, for 30l of water we need 20l of Dye \Rightarrow 8l of Dye should be added.

Hence, 8 is correct answer.

Question 24. The number of real-valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$ is

- A. infinite
- B. 0
- C. 1
- D. 2

Answer. B

Solution.

$$2^x + 2^{-x} = 2 - (x - 2)^2$$

$$\Rightarrow 2^x + 1/2^x = 2 - (x - 2)^2$$

\Rightarrow By putting options

$$\Rightarrow x = 1 \text{ then } 2^1 + 2^{-1} = 2 - (1 - 2)^2$$

$$\Rightarrow 2 + 0.5 = 2 - 1$$

\Rightarrow LHS is not equal to RHS

\Rightarrow when $x = 2$

$$\Rightarrow 2^2 + 1/2^2 = 2 - (2 - 2)^2$$

$$\Rightarrow 4 + 0.25 = 2$$

\Rightarrow LHS is not equal to RHS

\Rightarrow Hence this equation has 0 solution

Question 25. If $\log_4 5 = (\log_4 y) (\log_6 \sqrt{5})$, then y equals

Answer. 36

Solution.

According to the question,

$$\Rightarrow \log_4 5 = \log_4 y \times \log_6 \sqrt{5}$$

$$\Rightarrow (\log_4 5) / (\log_4 y) = \log_6 \sqrt{5} = \log_y 5 = k$$

$$\Rightarrow 6^k = \sqrt{5}$$

On squaring, we get

$$\Rightarrow (6^k)^2 = 5 \quad \dots(i)$$

$$\Rightarrow \log_4 5 = \log_4 y \times \log_6 \sqrt{5}$$

$$\Rightarrow \log_y 5 = \log_6 \sqrt{5}$$

$$\Rightarrow \log_y 5 = k$$

$$\Rightarrow y^k = 5 \quad \dots(ii)$$

From equation (i) and (ii)

$$\Rightarrow (6^k)^2 = y^k$$

$$\Rightarrow (6^2)^k = y^k$$

$$\Rightarrow (36)^k = y^k$$

On comparing

$$\Rightarrow y = 36$$

\therefore The y equals 36.

Question 26. In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to

- A. 59
- B. 62
- C. 66
- D. 55

Answer. C

Solution. Let the total number of people in the group be 100.

Then, the number of young people is 28, and the number of old people is $100 - 28 = 72$.

The number of literates is 65, and the number of illiterates is $100 - 65 = 35$.

The number of young literates is 25% of 65, which is $65 * 0.25 = 16.25$.

Therefore, the number of old literates is $65 - 16.25 = 48.75$.

The percentage of old people among the illiterates is:

$$(48.75 / 35) * 100 = 139.857\%$$

This is closest to 139.86%, which is between 139 and 140. Therefore, the answer is C. 66.