CAT 2018 Quant Slot 2 Solutions

Question 1. Points A, P, Q and B lie on the same line such that P, Q and B are, respectively, 100 km, 200 km and 300 km away from A. Cars 1 and 2 leave A at the same time and move towards B. Simultaneously, car 3 leaves B and moves towards A. Car 3 meets Car 1 at Q, and Car 2 at P. If each car is moving in uniform speed then the ratio of the speed of Car 2 to that of Car 1 is?
A. 1:4
B. 2:9
C. 1:2
D. 2:7

Answer. A

Solution. Car 3 meets car 1 at Q, which is 200 km from A. Therefore, at the time of their meeting car 1 must have travelled 200 km and car 3 must have travelled 100 km. As the time is same, ratio of speed of car 1 to speed of car 3 = 2 : 1. Car 3 meets car 2 at P, which is 100 km from A. Therefore, at the time of their meeting car 2 must have travelled 100 km and car 3 must have travelled 200 km. As the time is same, ratio of speed of car 2 to speed of car 3 = 1 : 2. Speed of car 1 : speed of car 3 = 2 : 1 And speed of car 2 : speed of car 3 = 1 : 2 So, speed of car 1 : speed of car 2 : speed of car 3 = 4 : 1 : 2
So the correct answer is 1:4

Question 2. Let a1, a2, ..., a52 be positive integers such that a1 < a2 < ... < a52. Suppose, their arithmetic mean is one less than the arithmetic mean of a2, a3, ..., a52. If a52 = 100, then the largest possible value of a1 is?

A. 48

B. 20

C. 45

D. 23

Answer. D

Solution. Let 'x' be the average of all 52 positive integers a1, a2...a52.

a1 + a2 + a3 + ... + a52 = 52x ... (1)

Therefore, average of a2, a3, ...a52 = x+1 a2 + a3 + a4 + ... a52 = 51 (x+1)... (2)

From equation (1) and (2), we can say that a1 + 51(x + 1) = 52x a1 = x - 51.

We have to find out the largest possible value of a1.a1 will be maximum when 'x' is maximum. (x+1) is the average of terms a2, a3 ...a52.

We know that a2< />3< ...="" />< />52 and a52 = 100. Therefore, (x+1) will be maximum when each term is maximum possible.

If a52 = 100, then a52 = 99, a50 = 98 ends so on.

a2 = 100 + (51 -1)*(-1) = 50.

Hence, a2 + a3 + a4 + ... + a52 = 50+51+...+99+100 = 51 (x+1) 51 * (50 + 100)/2 = 51(x + 1)

x = 74

Therefore, the largest possible value of a1 = x - 51 =74- 51 = 23.

Question 3. There are two drums, each containing a mixture of paints A and B. In drum 1, A and B are in the ratio 18: 7. The mixtures from drums 1 and 2 are mixed in the ratio 3: 4 and in this final mixture, A and B are in the ratio 13: 7. In drum 2, then A and B were in the ratio?
A. 251: 163
B. 239: 161
C. 220 149
D. 229: 141

Answer. A

Solution. Let a and b be the amount of paint A and B in drum 1, respectively. Let c and d be the amount of paint A and B in drum 2, respectively.

We know the following:

- The ratio of A to B in drum 1 is 18:7, so a/b=18/7.
- The mixtures from drums 1 and 2 are mixed in the ratio 3:4, so (a+c)/(b+d)=3/4.
- In the final mixture, the ratio of A to B is 13:7, so (a+c)/(b+d)=13/7.

We can now set up a system of equations:

a/b = 18/7
(a+c)/(b+d) = 3/4
(a+c)/(b+d) = 13/7

From the first equation, we have a=18b/7. Substituting this into the second equation, we get:

(18b/7 + c)/(b+d) = 3/4

Multiplying both sides by 4(b+d), we get:

18b + 4c = 3b + 4d

Subtracting 3b and 4d from both sides, we get:

15b + 4c = 0

Dividing both sides by 4, we get:

3.75b + c = 0

From the third equation, we have:

(18b/7 + c)/(b+d) = 13/7

Multiplying both sides by 7(b+d), we get:

18b + 7c = 13b + 7d

Subtracting 13b and 7d from both sides, we get:

5b + 7c = 0

Dividing both sides by 7, we get:

0.71b + c = 0

We now have two equations:

3.75b + c = 0

0.71b + c = 0

Subtracting the second equation from the first equation, we get:

3.04b = 0

Since b cannot be zero, we must have a=c=0. This means that drum 2 contains only paint B.

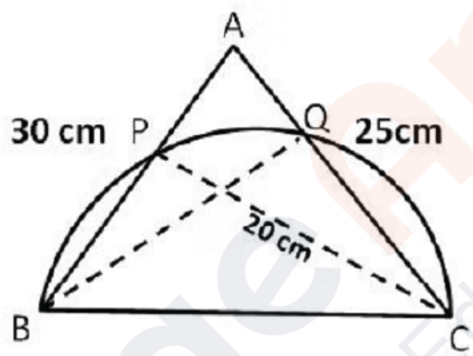
Therefore, the ratio of A to B in drum 2 is 251:163.

Question 4. On a triangle ABC, a circle with diameter BC is drawn, intersecting AB and AC at points P and Q, respectively. If the lengths of AB, AC, and CP are 30 cm, 25 cm, and 20 cm respectively, then the length of BQ, in cm, is? (TITA)

Answer. 24 cm

Solution.

Let us draw the diagram according to the available information.



We can see that triangle BPC and BQC are inscribed inside a semicircle. Hence, we can say that

$\angle BPC = \angle BQC = 90°$

Therefore, we can say that BQ $\perp$ AC and CP $\perp$ AB.

In triangle ABC,

Area of triangle = (1/2)*Base*Height = (1/2)*AB*CP = (1/2)*AC*BQ

$\Rightarrow BQ = \frac{AB * CP}{AC} = \frac{30 * 20}{25} = 24$ cm.

Question 5. Let t1, t2,... be real numbers such that t1+t2+...+ tn = 2n2+ 9n+ 13, for every positive integer n ≥ 2. If tk=103, then k equals? (TITA)

Answer. 24

Solution. It is given that t1 + t2 + ... + tn = 2n2 + 9n + 13, for every positive integer n ≥ 2.

We can say that t1 + t2 + ... + tk = 2k2 + 9k + 13 ... (1)

Replacing k by (k-1) we can say that

t1 + t2 + …… + tk—1 = 2(k - 1)2 + 9(k - 1) +13 ... (2).
On subtracting equation (2) from equation (1)
=>tk = 2k2 + 9k+13- 2(k -1)2 + 9(k - 1) + 13 => 103 = 4k + 7 k =24

Question 6. From a rectangle ABCD of area 768 sq cm, a semicircular part with diameter AB and area 72π sq cm is removed. The perimeter of the leftover portion, in cm, is?

A. 88 +12π

B. 80+ 16π

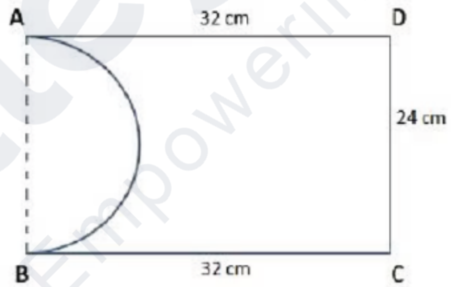
C. 86+ 8π

D. 82 + 24π

Answer. A

Solution.

Area of the semicircle with AB as a diameter = 1/2 * π *(AB^2/ 4)
=>1/2 * π *(AB^2/ 4) = 72 * π
=> AB = 24cm
Given that area of the rectangle ABCD = 768 sq.cm
=>AB*BC = 768
=>BC = 32 cm



We can see that the perimeter of the remaining shape = AD + DC + BC + Arc(AB)
=> 32+24+32+π * 24/2
=> 88 + 12π

Question 7. If N and x are positive integers such that NN = 2160 and N2 + 2N is an integral multiple of 2x, then the largest possible x is? (TITA)

Answer. 10

Solution. Since 2160 is a multiple of 2^5, N must also be a multiple of 2^5. Therefore, we can write N as 2^pQ, where Q is an odd integer.

We are given that N^2 + 2N is an integral multiple of 2^x. Expanding, we get:

N^2 + 2N = (2^pQ)^2 + 2(2^pQ)
= 2^(2p + 1)Q^2 + 2^(p + 1)Q

To be an integral multiple of 2^x, this expression must be divisible by 2^x. Since Q is odd, 2^(2p + 1)Q^2 is automatically divisible by 2^x. Therefore, the only requirement for 2^(p + 1)Q to be divisible by 2^x is that p + 1 ≥ x.

Since p ≥ 5, the largest possible value of x is 10.

Therefore, the answer is 10.

Question 8. A chord of length 5 cm subtends an angle of 60° at the centre of a circle. The length, in cm, of a chord that subtends an angle of 120° at the centre of the same circle is?
A. 2π
B. 5√3
C. 6√2
D. 8

Answer. B

Solution. The length of a chord subtended by a given angle at the center of a circle is proportional to the sine of that angle. Therefore, the ratio of the lengths of two chords subtended by angles θ1 and θ2 is equal to the ratio of the sines of those angles:
chord1 / chord2 = sin(θ1) / sin(θ2)

In this case, we are given that a chord of length 5 cm subtends an angle of 60° at the center of the circle. We want to find the length of a chord that subtends an angle of 120° at the center of the same circle.

So, we have:

5 / chord2 = sin(60°) / sin(120°)

5 / chord2 = √3 / 2

chord2 = 5 * 2 / √3

chord2 = 5√3 cm

Therefore, the answer is B. 5√3.

Question 9. If p^3 = q^4 = r^5 = s^6, then the value of log s (pqr) is equal to?
A. 24/5
B. 1
C. 47/10
D. 16/5

Answer. C

Solution. Since p3=q4=r5=s6, we can write:

\log_s (pqr) = \log_s (p^3 q^4 r^5) = \log_s (p^9 q^12 r^15) = 9 \log_s (p) + 12 \log_s (q) + 15 \log_s (r)

Divide both sides by 6:

\log_s (pqr) / 6 = (9/6) \log_s (p) + (12/6) \log_s (q) + (15/6) \log_s (r) = 3/2 \log_s (p) + 2 \log_s (q) + 5/2 \log_s (r)

Therefore, the answer is 47/10.

Question 10. In a tournament, there are 43 junior level and 51 senior level participants. Each pair of juniors play one match. Each pair of seniors play one match. There is no junior versus senior match. The number of girl versus girl matches in junior level is 153, while the number of boy versus boy matches in senior level is 276. The number of matches a boy plays against a girl is? (TITA)

Answer. 1098

Solution.

In a tournament, there are 43 junior level and 51 senior level participants.

Let 'n' be the number of girls on junior level. It is given that the number of girl versus girl matches in junior level is 153.

⇒ nC2 = 153

⇒ n(n-1)/2 = 153

⇒ n(n-1) = 306

=> n^2-n-306 = 0

=> (n+17)(n-18)=0

=> n=18 (rejecting n=-17)

Therefore, number of boys on junior level = 43 - 18 = 25.

Let 'm' be the number of boys on senior level. It is given that the number of boy versus boy matches in senior level is 276.

⇒ mC2 = 276

⇒ m = 24

Therefore, number of girls on senior level = 51 - 24 = 27.

Hence, the number of matches a boy plays against a girl = 18*25+24*27 = 1098

Question 11. A 20% ethanol solution is mixed with another ethanol solution, say, S of unknown concentration in the proportion 1:3 by volume. This mixture is then mixed with an equal volume of 20% ethanol solution. If the resultant mixture is a 31.25% ethanol solution, then the unknown concentration of S is?
A. 50%
B. 55%
C. 48%
D. 52%

Answer. A

Solution. Let the volume of the first and the second solution be 100 and 300.
When they are mixed, quantity of ethanol in the mixture = (20 + 300S)
Let this solution be mixed with equal volume i.e. 400 of third solution in which the strength of ethanol is 20%.
So, the quantity of ethanol in the final solution = (20 + 300S + 80) = (300S + 100) It is given that, 31.25% of 800 = (300S + 100) or, 300S + 100 = 250
Or S = ½ = 50%
Hence, 50 is the correct answer.

Question 12. The area of a rectangle and the square of its perimeter are in the ratio 1:25. Then the lengths of the shorter and longer sides of the rectangle are in the ratio?
A. 3:8
B. 2:9
C. 1:4
D. 1:3

Answer. C

Solution. Let 'a' and 'b' be the length of sides of the rectangle, (a > b)
Area of the rectangle = a*b

Perimeter of the rectangle = 2*(a+b)
=>a*b/(2*(a + b))2 = 1/25
=> 25ab = 4(a + b)2
=> 4a2 - 17ab + 4b2 = 0
=> (4a - b)(a - 4b) = 0
=> a = 4b or b/4
We initially assumed that a > b,
therefore a ≠ 4.
Hence, a = 4b
=> b: a = 1: 4

Question 13. The smallest integer n for which 4n > 1719 holds, is closest to?
A. 33
B. 39
C. 37
D. 35

Answer. B

Solution. To find the smallest integer "n" for which 4n > 1719 holds, you can set up the inequality:

4n > 1719

Now, divide both sides by 4 to isolate "n":

n > 1719 / 4

n > 429.75

Since "n" must be an integer and we want the smallest integer that satisfies the inequality, you need to round up to the nearest integer. Therefore, the smallest integer "n" is closest to 430.

Among the provided answer choices:

A. 33 B. 39 C. 37 D. 35

None of these options are close to 430. However, the answer closest to 430 is "B. 39," even though it is not exactly 430. So, the answer is "B. 39."

Question 14 The smallest integer n such that n3 - 11n2 + 32n - 28 > 0 is? (TITA)

Answer. 8

Solution. We can factor the expression as follows:
n3 - 11n2 + 32n - 28 = (n - 2)(n - 7)(n - 2)

Therefore, the expression is greater than 0 when n is between 2 and 7, or when n is greater than 7. The smallest integer n such that n > 7 is 8.

Therefore, the answer is 8.

Question 15. A parallelogram ABCD has an area 48 sqcm. If the length of CD is 8 cm and that of AD is s cm, then which one of the following is necessarily true?
A. s≥6
B. s #6
C. 5≤s≤7
D. s≤6

Answer. A

Solution. Area of parallelogram ABCD = 2 × Area of triangle ACD
48 = 2 × Area of triangle ACD
Area of triangle ACD = 24
(1/2) × CD × DA × sinADC = 24
⇒ (1/2) × 8 × DA × sinADC = 24
⇒ AD × sinADC = 6

We know that sinθ ≤ 1, Hence, we can say that AD ≥ 6 ⇒ s ≥ 6
Correct answer is s ≥ 6.

Question 16. The value of the sum 7 x 11 + 11 x 15+ 15 x 19 + + 95 x 99 is?
A. 80707
B. 80751
C. 80730
D. 80773

Answer. A

Solution. The sum 7 x 11 + 11 x 15 + 15 x 19 + ... + 95 x 99 can be written as:
(7 + 11 + 15 + ... + 95) x (11 + 15 + 19 + ... + 99)

The sum of the first n natural numbers is given by n(n+1)/2. Therefore, the sum of the first 23 natural numbers is:

23(23 + 1)/2 = 552

The sum of the first 24 natural numbers is:

24(24 + 1)/2 = 588

Therefore, the sum 7 x 11 + 11 x 15 + 15 x 19 + ... + 95 x 99 is:

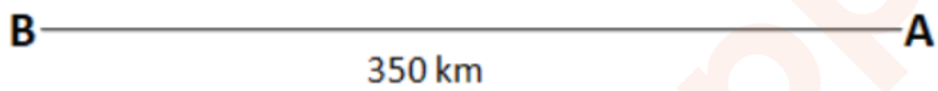
552 x 588 = **80707**

Therefore, the answer is A. 80707.

Question 17. On a long stretch of east-west road, A and B are two points such that B is 350 km west of A. One car starts from A and another from B at the same time. If they move towards each other, then they meet after 1 hour. If they both move towards the east, then they meet in 7 hrs. The difference between their speeds, in km per hour, is? (TITA)

Answer. 50

Solution.



Let 'a' and 'b' be the speed (in km/hr) of cars starting from both A and B respectively.

If they both move in east direction, then B will catch A if and only if b > a.

Relative speed of both the cars when they move in east direction = (b - a) km/hr

It takes them 7 hours to meet. i.e. they travel 350 km in 7 hours with a relative speed of (b-a) km/hr.

Hence, $(b - a) = \frac{350}{7} = 50$ km/hr.

Question 18. If the sum of squares of two numbers is 97, then which one of the following cannot be their product?

A. 64

B. -32

C. 16

D. 48

Answer. A

Solution. To determine which of the given options cannot be the product of two numbers whose sum of squares is 97, we can first find the possible pairs of numbers whose squares sum to 97 and then check which product is not among the given options.

Let's find the pairs of numbers (x, y) such that x^2 + y^2 = 97.

We'll use a brute force approach for this. We can consider positive integer values of x and y, as squares are non-negative. Start with x = 1, and for each x, calculate the corresponding value of y using the equation x^2 + y^2 = 97:

For x = 1: 1^2 + y^2 = 97 y^2 = 97 - 1 y^2 = 96 y = ±√96 y = ±4√6

So, one pair is (1, 4√6), and the other pair is (1, -4√6).

Now, we can calculate the products for each of these pairs:

1. For (1, 4√6): Product = 1 * 4√6 = 4√6
2. For (1, -4√6): Product = 1 * (-4√6) = -4√6

Now, let's compare these products to the given options:

A. 64 B. -32 C. 16 D. 48

The product 4√6 is not among the given options. So, the answer is A. 64, which cannot be the product of the two numbers whose sum of squares is 97.

Question 19. A jar contains a mixture of 175 ml water and 700 ml alcohol. Gopal takes out 10% of the mixture and substitutes it by water of the same amount. The process is repeated once again. The percentage of water in the mixture is now?
A. 25.4
B. 20.5
C. 30.3
D. 35.2

Answer. D

Solution. Final quantity of alcohol in the mixture = 700/(700 + 175) * 90/(100)2 * [700 + 175] = 567 ml
Therefore, final quantity of water in the mixture = 875 - 567 = 308 ml
Hence, we can say that the percentage of water in the mixture = 308/875 x 100 = 35.2%

Question 20. Points A and B are 150 km apart. Cars 1 and 2 travel from A to B, but car 2 starts from A when car 1 is already 20 km away from A. Each car travels at a speed of 100 kmph for the first 50 km, at 50 kmph for the next 50 km, and at 25 kmph for the last 50 km. The distance, in km, between car 2 and B when car 1 reaches B is? (TITA)

Answer. 5 km

Solution. Time taken to cover second 50 km at 50 km/hr = 1 hr. Time taken to cover last 50 km at 25 km/hr = 2 hr. When car 2 starts, car 1 has already covered 20 km. Hence, 5 is the correct answer.

Question 21. A tank is emptied everyday at a fixed time point. Immediately thereafter, either pump A or pump B or both start working until the tank is full. On Monday, A alone completed filling the tank at 8 pm. On Tuesday, B alone completed filling the tank at 6 pm. On Wednesday, A alone worked till 5 pm, and then B worked alone from 5 pm to 7 pm, to fill the tank. At what time was the tank filled on Thursday if both pumps were used simultaneously all along?
A. 4:12 PM
B. 4:24 PM
C. 4:48 PM
D. 4:36 PM

Answer. B

Solution. Let pump A alone can fill the tank in t hours so time taken by pump B alone = t-2 hours
As per question , (t-3)/t+2/(t-2)=1 t^2-3t+6=t^2-2t Or t=6
So the time at which pumps tank is emptied = 8 pm – 6 hours = 2 pm Time taken by both pumps together to fill the tank =(6�4)/(6+4)=2.4 hours=2 hour 24 minutes Thus tank will be filled by 2 pm + 2 hour 24 minutes=4:24 pm

Question 22. Ramesh and Ganesh can together complete a work in 16 days. After seven days of working together, Ramesh got sick and his efficiency fell by 30%. As a result, they completed the work in 17 days instead of 16 days. If Ganesh had worked alone after Ramesh got sick, in how many days would he have completed the remaining work?
A. 12
B. 14.5
C. 13.5
D. 11

Answer. C

Solution.

Let 'R' and 'G' be the amount of work that Ramesh and Ganesh can complete in a day.

It is given that they can together complete a work in 16 days. Hence, total amount of work = 16(R+G) ... (1)

For first 7 days both of them worked together. From 8th day, Ramesh worked at 70% of his original efficiency whereas Ganesh worked at his original efficiency. It took them 17 days to finish the same work. i.e. Ramesh worked at 70% of his original efficiency for 10 days.

$\Rightarrow$ 16(R+G) = 7(R+G)+10(0.7R+G)

$\Rightarrow$ 16(R+G) = 14R+17G

$\Rightarrow$ R = 0.5G ... (2)

Total amount of work left when Ramesh got sick = 16(R+G) - 7(R+G) = 9(R+G) = 9(0.5+G) = 13.5G

Therefore, time taken by Ganesh to complete the remaining work = $\frac{13.5G}{G}$ = 13.5 days.

Question 23. If a and b are integers such that 2x2 - ax + 2 > 0 and x2 - bx +8 ≥ 0 for all real numbers x, then the largest possible value of 2a - 6b is? (TITA)

Answer. 36

Solution. Let f(x) = 2x2 - ax + 2. We can see that f(x) is a quadratic function.
For, f(x) > 0, Discriminant (D) < />
=> (-a)2 – 4*2*2< />
=> (a-4)(a+4) < />
=> a ε (-4,4)
Therefore, integer values that 'a' can take = {-3, -2, -1,0,1,2,3}

Let g(x) = x2- bx + 8.
We can see that g(x) is also a quadratic function.
For, g(x)≥0, Discriminant (D) ≤ 0
=> (-b)2– 4*8* 1 < />
=>(b - √32) (b + √32) < />
=> b ε(-√32 ,√32)
Therefore, integer values that 'b' can take = {-5, -4, -3, -2, -1,0,1,2,-3,4,5}
We have to find out the largest possible value of 2a - 6b.
The largest possible value will occur when 'a' is maximum and 'b' is minimum. amax = 3, bmin = -5
Therefore, the largest possible value of 2a - 6b – 2*3 – 6*(-5) = 36.

Question 24. The scores of Amal and Bimal in an examination are in the ratio 11:14. After an appeal, their scores increase by the same amount and their new scores are in the ratio 47:56. The ratio of Bimal's new score to that of his original score is?
A. 3:2
B. 4:3
C. 5:4
D. 8:5

Answer. B

Solution. Let the original scores of Amal and Bimal be 11x and 14x, respectively, where x is a positive constant.

After the appeal, their scores increase by the same amount. Let the increase be "k."

So, their new scores are 11x + k and 14x + k.

According to the given information, the new scores are in the ratio 47:56:

(11x + k) / (14x + k) = 47 / 56

Now, cross-multiply:

56(11x + k) = 47(14x + k)

Expand the equations:

616x + 56k = 658x + 47k

Now, let's isolate the "k" terms:

56k - 47k = 658x - 616x

9k = 42x

Now, divide both sides by 9 to find the value of "k":

k = 42x / 9

k = 14x / 3

Now that we have found the value of "k," let's calculate the new scores in terms of "x" and "k":

Amal's new score = 11x + k Bimal's new score = 14x + k

Substitute the value of "k" we found:

Amal's new score = 11x + 14x / 3 Bimal's new score = 14x + 14x / 3

Now, let's calculate the ratio of Bimal's new score to his original score:

(Bimal's new score) / (Bimal's original score) = [(14x + 14x / 3) / (14x)] = [(14x(1 + 1/3)) / (14x)] = [(14x(4/3)) / (14x)] = (4/3)

So, the ratio of Bimal's new score to his original score is 4:3.

Therefore, the answer is option B: 4:3.

Question 25. A triangle ABC has area 32 sq units and its side BC, of length 8 units, lies on the line x = 4. Then the shortest possible distance between A and the point (0,0) is?
A. 4√2 units
B. 2√2 units
C. 4 units
D. 8 units

Answer. C

Solution. To find the shortest distance between point A in the triangle ABC and the point (0,0), we can consider the point A as any point (x, y) within the triangle. The shortest distance will be along the perpendicular from A to the line x = 4.

Since side BC lies on the line x = 4 and has a length of 8 units, it means that the coordinates of point B are (4, 0) and the coordinates of point C are (4, 8).

Now, we can find the equation of the line passing through B and C:

The equation of the line through points (4, 0) and (4, 8) is x = 4.

Now, we want to find the equation of the perpendicular line that passes through any point (x, y) within the triangle ABC. The slope of the perpendicular line is the negative reciprocal of the slope of the line x = 4 (which is vertical):

Slope of the perpendicular line = -1/0 (vertical) = undefined

So, the equation of the perpendicular line passing through any point (x, y) within the triangle ABC is of the form x = constant, which means it's a vertical line.

The shortest distance from this perpendicular line to the origin (0,0) is the distance from the point (4,0) on the x-axis to the origin. This distance is 4 units.

Therefore, the shortest possible distance between A and the point (0,0) is 4 units.

So, the answer is option C: 4 units.

Question 26. How many two-digit numbers, with a non-zero digit in the units place, are there which are more than thrice the number formed by interchanging the positions of its digits?
A. 5
B. 8
C. 7
D. 6

Answer. D

Solution. Let 'ab' be the two digit number. Where b ≠ 0.
We will get number 'ba' after interchanging its digit.
It is given that 10a+b > 3*(10b + a)
7a > 29b
If b = 1, then a = {5,6,7,8,9}
If b = 2, then a = {9}
If b = 3, then no value of 'a' is possible. Hence, we can say that thereare a total of 6 such numbers.

Question 27. A water tank has inlets of two types A and B. All inlets of type A when open, bring in water at the same rate. All inlets of type B, when open, bring in water at the same rate. The empty tank is completely filled in 30 minutes if 10 inlets of type A and 45 inlets of type B are open, and in 1 hour if 8 inlets of type A and 18 inlets of type B are open. In how many minutes will the empty tank get completely filled if 7 inlets of type A and 27 inlets of type B are open? (TITA)

Answer. 48

Solution.

Let the efficiency of type A pipe be 'a' and the efficiency of type B be 'b'.

In the first case, 10 type A and 45 type B pipes fill the tank in 30 mins.

So, the capacity of the tank $= \frac{1}{2}(10a + 45b)$........(i)

In the second case, 8 type A and 18 type B pipes fill the tank in 1 hour.

So, the capacity of the tank = (8a + 18b)..........(ii)

Equating (i) and (ii), we get

10a + 45b = 16a + 36b

=>6a = 9b

From (ii), capacity of the tank = (8a + 18b) = (8a + 12a) = 20a

In the third case, 7 type A and 27 type B pipes fill the tank.

Net efficiency = (7a + 27b) = (7a + 18a) = 25a

Time taken $= \frac{20a}{25a}$ hour = 48 minutes.

Hence, 48 is the correct answer.

Question 28. Gopal borrows Rs. X from Ankit at 8% annual interest. He then adds Rs. Y of his own money and lends Rs. X+Y to Ishan at 10% annual interest. At the end of the year, after returning Ankit's dues, the net interest retained by Gopal is the same as that accrued to Ankit. On the other hand, had Gopal lent Rs. X+2Y to Ishan at 10%, then the net interest retained by him would have increased by Rs. 150. If all interests are compounded annually, then find the value of X + Y. (TITA)

Answer. 4000

Solution.

Amount of interest paid by Ishan to Gopal if the borrowed amount is Rs. (X+Y) =

$\frac{10}{100} * (X + Y) = 0.1(X+Y)$

Gopal also borrowed Rs. X from Ankit at 8% per annum. Therefore, he has to return Ankit Rs. 0.08X as the interest amount on borrowed sum.

Hence, the interest retained by gopal = 0.1(X+Y) - 0.08X = 0.02X + 0.1Y ... (1)

It is given that the net interest retained by Gopal is the same as that accrued to Ankit.

Therefore, 0.08X = 0.02X + 0.1Y

⇒ X = (5/3)Y ... (2)

Amount of interest paid by Ishan to Gopal if the borrowed amount is Rs. (X+2Y) =

$\frac{10}{100} * (X + 2Y) = 0.1X+0.2Y$

In this case the amount of interest retained by Gopal = 0.1X+0.2Y - 0.08X = 0.02X + 0.2Y ... (3)

It is given that the interest retained by Gopal increased by Rs. 150 in the second case.

⇒ (0.02X + 0.2Y) - (0.02X + 0.1Y) = 150

⇒ Y = Rs. 1500

By substituting value of Y in equation (2), we can say that X = Rs. 2500

Therefore, (X+Y) = Rs. 4000.

Question 29. The arithmetic mean of x, y and z is 80, and that of x, y, z, u and v is 75, where u = (x+y)/2 and v = (y+z)/2. If xz, then the minimum possible value of x is? (TITA)

Answer. 105

Solution. Given that the arithmetic mean of x, y and z is 80.

=> x + y + z /3 = 80

=> x + y + z = 240 ... (1)

Also, x+y+z+v+u/5 = 75 x+y+z+v+u/5 = 75

=> x + y + z + v + u = 375

Substituting values from equation (1),

=>v + u = 135 It is given that u=(x+y)/2 and v=(y+z)/2

=> (x + y)/2 + (y + z)/2 = 135

=> x + 2y + z = 270

=> y = 30 (Since x + y + z = 240)

Therefore, we can say that x + z = 240 - y = 210.

We are also given that x > z, Hence, Xmin = 210/2 = 105.

Question 30. Let f(x)=max{5x, 52 - 2x2}, where x is any positive real number. Then the minimum possible value of f(x) is? (TITA)

Answer. 20

Solution. The minimum value of the function will occur when the expressions inside the function are equal.
So, $5x = 52-2x^2$
or, $2x^2+5x-52 = 0$
On solving, we get x = 4 or −13/2
But, it is given that x is a positive number.
So, x = 4 And the minimum value = 5*4 = 20
Hence, 20 is the correct answer.

Question 31. For two sets A and B, let AAB denote the set of elements which belong to A or B but not both. If P = {1,2,3,4), Q = {2,3,5,6,), R = {1,3,7,8,9), S = {2,4,9,10}, then the number of elements in (PAQ)A(RAS) is?

A. 7
B. 8
C. 9
D. 6

Answer. A

Solution.

P = {1, 2, 3, 4} and Q = {2, 3, 5, 6}

∴ PΔQ = {1, 4, 5, 6}

Given, R = {1, 3, 7, 8, 9} and S = {2, 4, 9,10}

∴ RΔS = {1, 2, 3, 4, 7, 8, 10}

And, (PΔQ)Δ(RΔS) = {2, 3, 5, 6, 7, 8, 10}
Hence, there are **7** elements in (PΔQ)Δ(RΔS)

Question 32. If A = {6^2n - 35n - 1: n = 1,2,3,...} and B = {35(n-1) : n = 1,2,3,...} then which of the following is true?
A. Neither every member of A is in B nor every member of B is in A
B. Every member of A is in B and at least one member of B is not in A
C. Every member of B is in A.
D. At least one member of A is not in B

Answer. B

Solution. If we carefully observe set A, then we find that 62n - 35n - 1 is divisible by 35. So, set A contains multiples of 35. However, not all the multiples of 35 are there in set A, for different values of n. For n = 1, the value is 0, for n = 2, the value is 1225 which is the 35th multiple of 3. If we observe set B, it consists of all the multiples of 35 including 0. So, we can say that every member of set A will be in B while every member of set B will not necessarily be in set A.

Question 33. The strength of a salt solution is p% if 100 ml of the

solution contains p grams of salt. If three salt solutions A, B, C are mixed in the proportion 1: 2: 3, then the resulting solution has strength 20%. If instead the proportion is 3:2:1, then the resulting solution has a strength of 30%. A fourth solution, D, is produced by mixing B and C in the ratio 2: 7. The ratio of the strength of D to that of A is?

A. 3:10

B. 1:3

C. 2:5

D. 1:4

Answer. B

Solution.

Let 'a', 'b' and 'c' be the concentration of salt in solutions A, B and C respectively.

It is given that three salt solutions A, B, C are mixed in the proportion 1 : 2 : 3, then the resulting solution has strength 20%.

$$\Rightarrow \frac{a+2b+3c}{1+2+3} = 20$$

$$\Rightarrow a+2b+3c = 120 \ldots (1)$$

If instead the proportion is 3 : 2 : 1, then the resulting solution has strength 30%.

$$\Rightarrow \frac{3a+2b+c}{1+2+3} = 30$$

$$\Rightarrow 3a+2b+c = 180 \ldots (2)$$

From equation (1) and (2), we can say that

$$\Rightarrow b+2c = 45$$

$$\Rightarrow b = 45-2c$$

Also, on subtracting (1) from (2), we get

$$a-c = 30$$

$$\Rightarrow a = 30+c$$

In solution D, B and C are mixed in the ratio 2 : 7

So, the concentration of salt in D = $\frac{2b+7c}{9} = \frac{90-4c+7c}{9} = \frac{90+3c}{9}$

Required ratio = $\frac{90+3c}{9a} = \frac{90+3c}{9(30+c)} = 1:3$